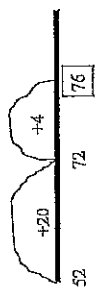


add
sums
to
start
with
smaller
numbers

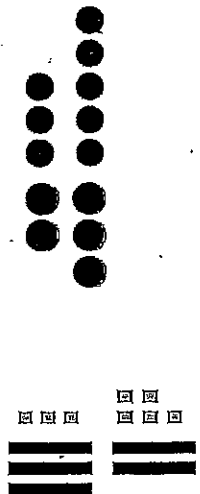
Stage 2
Pupils continue to use the number line to calculate with bigger numbers, partitioning the smaller number and adding the most significant digit first.

(c) $52 + 24 =$ 76
 (b) $63 + 14 =$ 77

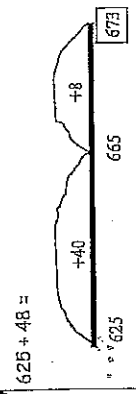


(c) $12 + 7 + 4 =$ 23

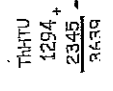
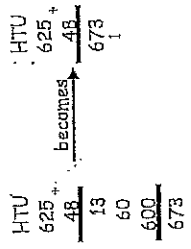
When children have a good understanding of place value and partitioning, introduce the columnar methods with additions that do not cross the tens boundary using concrete apparatus laid out in a columnar form.



Stage 4
Partition one number when adding mentally.

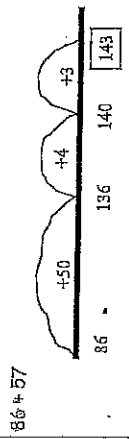


Pupils use their understanding of the expanded columnar methods of addition to progress to use the compact method.



+
+
+
Addition
+
+
algebra

Stage 3
Pupils continue to use the number line to support mental calculations.

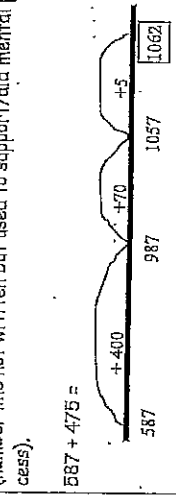


Pupils build on their understanding of place value, partitioning and their concrete experiences to develop columnar methods of addition which bridge the tens, then hundreds, initially in the expanded form.

Partitioning method

It is important that the children have a good understanding of place value and partitioning using concrete resources and visual images to support calculations. The expanded method enables children to see what happens to numbers in the standard written method.

Stage 5
Adding larger numbers mentally, partitioning the smaller numbers. (number line not written but used to support/aid mental process).

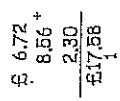


Number line used as a written method when working with time.

e.g. 46 minutes + 29 minutes =



Pupils use the compact column method to calculate with decimal numbers and with larger whole numbers.



increase
more
count on
more than

Stage 3 cont...

TU	HTU
67 +	83 +
24	42
11	5
80	120
91	125

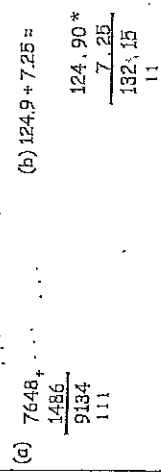
Progressing to 3-digit numbers.

HTU
124 +
137
11
50
200
251

Stage 6
Adding larger numbers mentally, supported by the number line, partitioning the smaller number. (As with stage 5-- number line not written but used to support/aid mental process).

Pupils should be using estimation skills to approximate an answer prior to using written method of addition.

Pupils add larger whole numbers using the columnar method. They add decimals with different numbers of decimal places using the columnar method. Pupils may fill 'empty' columns with zeros (0) and also add column headings initially to support and preserve place value.

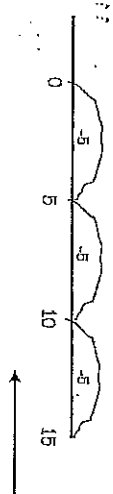


Page 2 Pupils calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (\times), division (\div) and equals ($=$) signs.

$4 \times 3 = 12$
 $3 \times 4 = 12$
 $12 \div 3 = 4$
 $12 \div 4 = 3$

Pupils solve problems involving multiplication and division, using practical materials, arrays, repeated subtraction, mental methods and multiplication and division facts, including problems in contexts.

Eg. 15 pencils are put into boxes of 5. How many boxes of pencils will there be?



There will be 3 boxes of 5 pencils.

Arrays

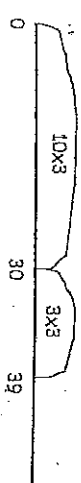


Also use arrays to model division.
 $15 \div 5 = 3$ and $15 \div 3 = 5$

Page 3 Pupils write and calculate mathematical statements for division using the multiplication tables that they know, using mental and progressing to formal written methods.

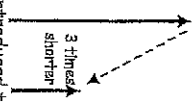
Use knowledge of multiplication facts and repeated addition to answer division questions.

Eg. How many 3s are there in 39?

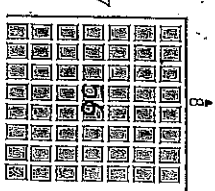


Extending to use all tables that pupils know to explore the idea of the remainder.

Stage 3 Pupils explore the use of scaling as a model for division, eg. My ribbon is 24cm long, can you cut a ribbon 3 times shorter?



Pupils are introduced to the formal written method of short division with whole number answers, using the image of the array and place value apparatus initially.



Pupils progress to use the formal written method of short division.

$$\begin{array}{r} 14 \\ 7 \overline{) 98} \end{array}$$

Stage 5

Pupils divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context, eg. 432 school children go on a camping trip. Each tent sleeps five. How many tents will they need to take?

432 \div 5 becomes:

$$\begin{array}{r} 86 \text{ r}2 \\ 5 \overline{) 432} \end{array}$$

Answer: 86 remainder 2
 Answer: they will need to take 87 tents.

Handwritten notes: *divide*, *share*, *split*, *groups of*, *square root*, *remainder*, *divisible*, *by*. A cloud contains the word **Division**.

Stage 6

Pupils divide numbers up to 4 digits by a 2-digit whole number using the formal written method of long division and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context, eg. Chocolates are packed in trays of 15. If I have 432 chocolates, how many full trays will I have and how many chocolates will be left over?

432 \div 15 becomes

$$\begin{array}{r} 28 \\ 15 \overline{) 432} \\ \underline{300} \\ 132 \\ \underline{120} \\ 120 \\ \underline{120} \\ 0 \end{array}$$

Answer: There will be 28 trays of chocolates and 12 chocolates left.

Pupils progress to expressing their remainders as a fraction. Eg. 432 litres of water are stored in 15 litre drums. How many full drums of water will there be and what fraction of the final drum will be filled?

432 \div 15 becomes

$$\begin{array}{r} 28 \\ 15 \overline{) 432} \\ \underline{300} \\ 132 \\ \underline{120} \\ 120 \\ \underline{120} \\ 0 \end{array}$$

Answer: There will be 28 full drums and the fifth drum will be $4/5$ full.

Stage 6 cont...

Progressing to expressing the remainder as a decimal eg. 5432 was raised at the school fair and is to be shared equally between 15 classes. How much will each class receive?

432 \div 15 becomes

$$\begin{array}{r} 28.8 \\ 15 \overline{) 432.0} \\ \underline{300} \\ 132 \\ \underline{120} \\ 120 \\ \underline{120} \\ 0 \end{array}$$

Answer: 28.8

Stage 4

Pupils continue to use the number line to support mental division.

Extend to 3-digit divided by a 1-digit number, 257 \div 7

Estimate first, use a number line to count on, if appropriate.



Pupils continue to become fluent with the formal written method of short division with exact answers. Eg.

$$\begin{array}{r} 23 \\ 6 \overline{) 138} \end{array}$$

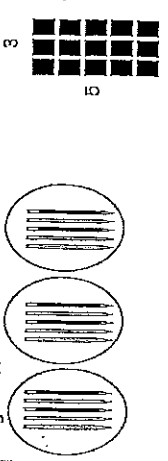
double
lots of
groups
multiple

Stage 2
Pupils calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (x), division (÷) and equals (=) signs.

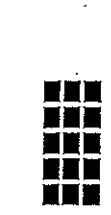
$2 \times 5 = 10$
 $12 = 4 \times 3$

Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods and multiplication and division facts, including problems in contexts, e.g.

3 friends have 5 pencils each. How many pencils do they have altogether?



$5 \times 3 = 15$ multiplied by 3 or 5 times 3 or 5, three times



Stage 4
Pupils multiply two-digit and three-digit numbers by a one-digit number using formal written layout

HTUxU using grid method, e.g. 136×5

x	100	30	6
5			

Progressing to the expanded short multiplication method (least significant digit first).

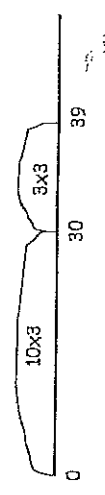
$$\begin{array}{r} 136 \\ \times 5 \\ \hline 30 \\ 150 \\ + 500 \\ \hline 680 \end{array}$$

Stage 4
Moving to the formal written method.

$$\begin{array}{r} 136 \\ \times 5 \\ \hline 680 \end{array}$$

Multiplication

Stage 3
Build on their understanding of repeated addition and arrays to multiply two digits by one digit using tables they know, e.g. 13×3



Informal recording of partitioned numbers,
 $15 \times 5 = 10 \times 5 + 5 \times 5$ or $10 \times 5 + 5 \times 5$

Stage 5
Multiply numbers up to 4-digits by a one-digit or two-digit number using a formal written method, including long multiplication for two-digit numbers.

Multiply THHTU x U using the formal written method, e.g. 1345×6

$$\begin{array}{r} 1345 \\ \times 6 \\ \hline 8070 \\ 223 \end{array}$$

Multiply TUXTU using the grid method, e.g. 38×72

x	30	8
70	2100	560
2	60	16
		2736

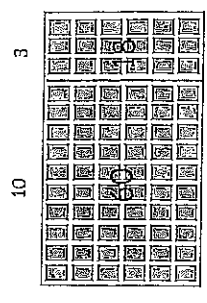
Progressing to the expanded written form for TU x TU

$$\begin{array}{r} 123 \\ \times 25 \\ \hline 615 \text{ (x5)} \\ 2460 \text{ (x20)} \\ \hline 3075 \end{array}$$

Extending to the formal written method of long multiplication (see stage 6).

times
multiply
array
factor
product
squared

Link arrays to introduce grid multiplication to multiply TU by U, e.g. 13×6



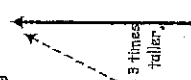
$60 + 18 = 78$

$13 \times 6 = 78$

Use grid methods to multiply TU by U, progressing to formal written methods when appropriate. (see stage 4).

x	20	3
4	80	12

Scaling



Relate multiplication to scaling.
My string is 12cm long. Cut a piece of string 3 times longer.

Stage 6
Pupils multiply multi-digit numbers up to 4-digits by a two-digit whole number using the formal written method of long multiplication.

124×26 becomes

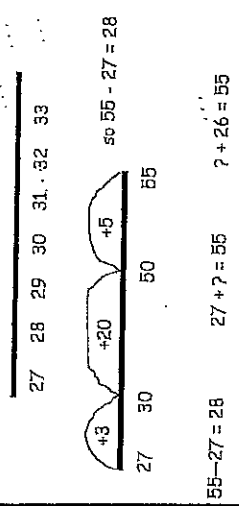
$$\begin{array}{r} 124 \\ \times 26 \\ \hline 744 \\ 2480 \\ \hline 3224 \end{array}$$

Answer: 3224

Minds
Left
to
work
differently

Stage 2
Pupils practise finding the difference by counting on using a number line. They are able to choose when to subtract and when to find the difference when answering a subtraction problem.

The difference between 27 and 34 is:



Pupils use concrete apparatus to explore exchange in practical activities e.g. subtract 18p from 33p



Pupils begin to organise their subtractions using expanded columnar methods.

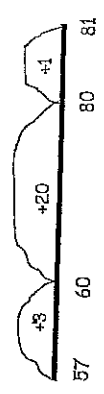
$$\begin{array}{r} 87 - 54 = \\ - 50 \quad 7 \\ \quad \quad 5 \\ \hline 30 \quad 2 \end{array}$$

Subtraction

leaves
cant
back
less than

Stage 3
Finding the difference by counting on.

$$81 - 57 =$$



Subtraction (take away)

$$\begin{array}{r} 81 - 57 = \\ - 57 \quad 11 \\ \quad \quad 4 \\ \hline 20 \quad 4 \\ = 24 \end{array}$$

70 11
80 4
50 7

Check answers with inverse.

Stage 3 cont...
Pupils progress to subtract numbers with up to 3-digits

$$341 - 123$$

$$\begin{array}{r} 300 \quad 40 \quad 11 \\ - 100 \quad 20 \quad 3 \\ \hline 200 \quad 10 \quad 8 \end{array}$$

Stage 4
Pupils continue to calculate difference mentally using a number line to support (does not have to be written—used to aid mental working).

Pupils progress to using the compact columnar method for subtraction.

$$\begin{array}{r} 784 = \\ - 56 \\ \hline 700 \quad 20 \quad 8 = 728 \end{array}$$

70 14
80 4
50 6

Adjust from T and U

Progressing to 4-digit numbers.

$$\begin{array}{r} 2754 = \\ - 1562 \\ \hline 1192 \end{array}$$

600 150
700 700 58 4
1000 500* 60 2

Stage 5
Pupils continue to calculate difference mentally using a number line to support (does not have to be written—used to aid mental working).

Pupils use the column method (standard method of subtraction) to solve increasingly more complex calculations involving many exchanges and solve subtractions with more than 4 digits.

$$\begin{array}{r} 531.04 \\ - 6467 \\ \hline 2684 \\ 3783 \end{array}$$

Stage 5 Cont...
Pupils subtract decimals with more than one decimal place and with differing numbers of digits.

$$\begin{array}{r} 324.9 \\ - 7.25 \\ \hline 317.65 \end{array}$$

Stage 6
Pupils continue to calculate difference mentally, using a number line to support, with increasingly large numbers (does not have to be written—used to aid mental working).

$$6467 - 2684$$

$$\begin{array}{r} 6467 \\ - 2684 \\ \hline 3783 \end{array}$$

and then check answers

$$3783 + 2684 = 6467$$

$$324.9 - 7.25$$

$$\begin{array}{r} 324.90 \\ - 7.25 \\ \hline 317.65 \end{array}$$

Continue to use the inverse to check.